A Simple Model of Pricing for Non-Storable Goods in Oligopoly

Some Considerations on Airline Pricing Behaviour

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Motivating Example: Theory

Market

A market (a single OD) is characterized by two cohorts of consumers: leisure ($t = 1$) and business ($t = 2$) travellers. Inverse demand: $P_t(D_t) = P(r_t, s_t; D_t) = r_t(1 - D_t/s_t)$, r_t is the maximal evaluation for a flight, and $r_1 < r_2$.

Monopoly (Text-book solution)

Assuming unit costs are c, then $p_t^m = \frac{1}{2}(r_t + c)$. Numerically, if $r_1 = 400$, $r_2 = 700$ and $c = 100$ then $p_1^m = 250$ and $p_2^m = 400$.

Duopoly (Text-book solution)

Bertrand: $p_t^b = c$, i.e: $p_1^m = p_2^m = 100$.
Cournot: $p_t^m = \frac{1}{3}(r_t + 2c)$, i.e. $p_1^c = 200$ and $p_2^c = 300$.

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Motivating Example: empirical evidence

Average fares per class of service[a]

Class of service	Monopoly	Duopoly
Promotional	183	169
Discounted1	305	266
Discounted2	395	350
Economy1	490	441
Economy2	607	557
Unrestricted1	893	780
Unrestricted2	977	883

[a] Sample of 15 OD destinations a 4 legacy carriers in Europe, round-trip flights, 2003(euros), Alderighi et al. (2004)

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A new version of Bertrand Paradox

Even if firms compete in prices (Bertrand), we observe a price patterns as if firms compete in quantity (Cournot)

1 cohort game, Kreps and Scheinkman (1983)

KS solve the Bertrand Paradox: They demonstrate that capacity precommitment and price competition lead to Cournot outcomes. Key element: costly or limited capacity

Goal of the paper

- To solve the Bertrand Paradox for N-Cohort Game
- To show why price patterns in monopoly and duopoly are very similar

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The model: structure of the market

- Z units of a non-storable good are produced at time 0 and they will be available at time T.
- For $t \in \mathcal{T} = \{1, .., T\}$, different cohorts of consumers become potentially interested in buying the good offered on the market.
- The demand is: $P_t(D_t) = r_t(1 - D_t/s_t)$, where:
  D_t=quantity demanded by cohort $t \in \mathcal{T}$,
  r_t=the maximal willingness-to-pay of consumers of cohort t,
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The model: supply

- There are two firms, named A and B. Firms sustain a cost c for each unit produced at time 0, but zero cost in selling the product at time $t \in \mathcal{T}$.
- Let be X and Y the production of firms A and B, respectively; and $X + Y = Z$. Firms are free to choose the quantity offered and the price charged at any time t.
- Let x_t and p_t be, respectively, the quantity and the price offered by firm A at time t.
- Similarly definitions apply to y_t and q_t for firm B.

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The model: demand

1. (Increasing consumer valuation) $r_s < r_t < \infty$, for $s < t$ and $s, t \in \mathcal{T}$.
2. (Efficient rationing rule) Consumers with the highest willingness-to-pay are first to be served.
3. (One visit, at the most) If a consumer belonging to the cohort t is not served, s/he exits the market (i.e. change the date, the destination, the way of transport, stay at home, etc..).
4. (Certainty) The demand is certain.
5. (Viable and unlimited demand for ε prices). $P_T(0) = r_T > c$, $P_1 = \varepsilon$ with $\varepsilon \in (0, c)$, i.e. $r_1 = \varepsilon$ and $s_1 = \infty$.

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The model: timing of the game

Case I

- (Demand) Nature determines the demand for each cohort.
- (Capacity choice) At time 0, firms choose X and Y, simultaneously.
- (Allocation choice for cohort t) At time 0.5, firms choose $(x_t)_{t\in\mathcal{T}}$ and $(y_t)_{t\in\mathcal{T}}$, simultaneously.
- (Pricing game for cohort t) At time $1, 2, ..T$, firms enter the pricing game, where p_t and q_t are simultaneously determined.

Case II

- (Demand) *The same.*
- (Capacity choice) *The same.*
- (Allocation choice for cohort t) At time $t - 0.5$, firms choose x_t and y_t, simultaneously.
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The model: Pricing game

Market-clearing competition

Similar to Cournot model. At time t, firms supply their quantities to consumers and then a fictional auctioneer computes the market-clearing price. In this case: $p_t = q_t = P_t(z_t)$, where $z_t = x_t + y_t$.

Bertrand-Edgeworth competition, KS (1983)

Firms choose prices, mixed strategies are allowed. Note: For some x_t and y_t, there is no equilibrium in pure strategies so that a direct computation of the equilibrium of the overall game is very complex.

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Solution strategy

To find the equilibrium outcome under market-clearing competition, and then to show that it is also an equilibrium outcome under Bertrand-Edgeworth competition (at least for some special cases). In particular:

- we solve the a market-clearing game, first when capacities X and Y are given, and then when carriers are free to choose.
- we test whether the outcome of the market-clearing game is olso an outcome of Bertrand-Edgeworth competition in three cases:
  1. 2 Cohorts
  2. Local Nash equilibrium
  3. Simulation

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Market-clearing competition: Monopoly

Proposition

If Z is given, $p_t^m = \frac{1}{2}(r_t + \lambda)$, and $z_t^m > 0$ if $r_t > \lambda$ and $z_t^m = 0$ if $r_t < \lambda$, where λ is the shadow price of capacity.

Therefore:

1. the monopolist will supply only those segments with the highest willingness-to-pay.
2. Being the monopolist free to move capacity from one cohort to the other, marginal revenue will be equalized among the active cohorts:

$$MR_t(z_t) = \lambda \text{ for those } t \text{ such that } r_t > \lambda$$

3. The optimal capacity choice Z^m is when $c = \lambda$.

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Market-clearing competition: Duopoly

Proposition

if firms A and B have given capacities $X, Y > 0$ with $X \geq Y$,then prices are given by:

$$p_t = q_t = \begin{cases} \frac{1}{3}\left(\lambda_x + \lambda_y + r_t\right) & t \geq t_B \\ \frac{1}{2}\left(r_t + \lambda_x\right) & t_A \leq t < t_B \\ \text{free} & t < t_A \end{cases}$$

where λ_x and λ_y are the shadow prices of expanding capacity, and $t_A \leq t_B : t_A = \min_t \{t : x_t > 0\}$, $t_B = \min_t \{t : y_t > 0\}$.

Market-clearing competition: Duopoly

Note

Firms' behaviour in duopoly is similar to that in monopoly

Therefore:

1. firms will supply only those segments with the highest willingness-to-pay.
2. Firms move capacity from one cohort to the other in order to equalize marginal revenue among active cohorts:

$$
\begin{aligned}
MR_t^A(x_t + y_t) &= \lambda_x \text{ for those } t \text{ such that } MR_t^A(y_t) > \lambda_x, \\
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3. The optimal capacities X^d and Y^d are given when $c = \lambda_x = \lambda_y$.

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Market-clearing competition: Duopoly, a note

Anderson and Fischer (1989) noted that the emergence of Cournot outcomes relies on the hypothesis of linear demand. Assuming different functional forms usually yield to different results. Anderson and Fischer (1989) have showed that a deviation from the Cournot capacity occurs when two simultaneous conditions realize: first, firms 'wish' to modify capacities, and, second, firms have the 'ability' to induce a change in the quantity supplied by the opponent. In our setup, the first condition is satisfied since some markets are more profitable than others ($r_t \neq r_s$ with $t \neq s$), but the second condition does not hold due to the linearity of demand.

Bertrand-Edgeworth competition: some results

Proposition: two-cohorts

For $T = 2$, the equilibrium outcome of the market-clearing competition is an equilibrium outcome of the game under Bertrand-Edgeworth competition.

Intuition: since market 1 will not be served in equilibrium, the model is very close to KS (1983)

Proposition: local Nash equilibrium

For every T, the equilibrium outcome of the market-clearing competition is a local equilibrium outcome of the game under Bertrand-Edgeworth competition.

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Bertrand-Edgeworth competition: simulation

Simulation steps:

1. Compute the optimal values of x_t^c, y_t^c, X^c and Y^c, for $c = 0.1$, $r_1, r_2 \in [1, 3]$ and $s_1, s_2 \in [1, 3]$
2. Change the capacity of B from 0 to $2Y$.
3. (For each capacity choice of B), compute the optimal allocation for A and B and the relative profits.
4. Compare the profit of B under deviation with the profit when B chooses Y^c

Simulation results

For $T = 2$, but without Assumption 5 (in order to have two standard linear demand functions) the equilibrium outcome of the market-clearing competition is an equilibrium outcome of the game under Bertrand-Edgeworth competition.

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Carriers behaviour: a question

Motivation
Linking theory & evidence
The model
Timing
Pricing Game
Solution strategy
Market-clearing competition
Bertrand-Edgeworth
Carriers behaviour
Conclusions

Why carriers will sell tickets to cohorts of consumers with lower willingness-to-pay without waiting for higher-valuation consumers?

The result rests on the type of competition. If carriers remain with large capacity for higher classes, they will not able to charge high fares to these cohorts of consumers, since pricing competition produces Bertrand-like results. On the contrary, by allocating part of their capacity to the lower classes, they are able to remain with the right capacity to obtain a Cournot-like result.

Carriers behaviour: some points

1. pricing patterns in duopoly mimic those in monopoly
2. If capacities are set once and for all, they can sustain a pricing pattern that is similar to empirical observations.
3. when capacities are fixed (f.e. as carrier do not change the size and capacity every day), carriers supply all travellers when there is low demand and only business travellers when demnad is high. (Systematic peak-load pricing, Borenstein and Rose, 1994)

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Conclusions

- This paper has presented a simple model of pricing that allows to explain some regularities observed in the airlines behaviour:
  1. firms intertemporal price discriminate and price levels increase in approaching the departure date,
  2. the pricing structure in oligopoly mimics that of monopoly case, although business and leisure price levels are lower,
  3. in low-demand periods, if firms are not allowed to re-size their planes, they offer discounted fares to very-low-valuation consumers (that usually do not purchase).
- It show that these results are due to the nature of competition. Even if firms compete in prices, capacity constrains limit the Bertrand-like outcomes, making the model predictions in line with Cournot setup.

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- The paper also provides a justification of the use of revenue management techniques (usually developed in monopoly) in oligopolistic markets, by making the role of capacity explicit.
- The model is based on many ad hoc assumptions, and some of these have important effects on the results.
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A Simple Model of Pricing for Non-Storable Goods in Oligopoly

Some Considerations on Airline Pricing Behaviour

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