A simple model of pricing for non-storable goods in oligopoly: Some considerations on airline pricing behaviour - Preliminary version

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Abstract

This paper provides a simple model of pricing (and capacity choice) for highly perishable goods, assuming that installing capacity is costly and high-valuation consumers arrive late. We show that oligopolistic firms find it optimal to engage in intertemporal price discrimination even if there is no uncertainty concerning the arrivals and there is no product differentiation. Indeed, firms are interested to sell a share of their production to low-valuation consumers at low prices to reduce their capacity in order to be able to charge high prices to high-valuation consumers. The outcomes of the model suitably describe the pricing behaviour observed in the airline industry.

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Keywords: Price discrimination; Oligopoly; Costly capacity; Airline industry.

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1 Introduction

Intertemporal price discrimination (IPD) is a widespread practice in airline and in many other retail industries (Borenstein, 1989; Borenstein and Rose, 1994; Hayes and Ross, 1998; Stavins, 2001). It consists to charge different prices for similar products, being price differentials not explained by cost variations (Clerides, 2004).[1] In oligopolistic contexts, scholars usually suggest that IPD is usually explained in terms of search costs or product differentiation (Krouse, 1990; Chp. 7). Search costs limit the number of stores a consumer can visit, reason why firms charging high prices can stay on the market (McMillan and Rothschild, 1994). Search theory explains price dispersed equilibria with many firms, but it is less convincing when the number of sellers is low. For the rest, product differentiation explains price discrimination even in duopoly (Holmes, 1989; Katz, 1984; Armstrong and Vickers, 2001; Rochet and Stole, 2002; Dessein, 2003; Alderighi, 2007).

In disagreement to previous explanations, Dana (1998,1999) showed that intertemporal price discrimination can result in absence of product differentiation by assuming demand uncertainty and costly capacity.

This work shows that IPD in absence of product differentiation may also be explained in a different way. More precisely, the paper presents a simple model of pricing (and capacity choice) for highly perishable goods, assuming that installing capacity is costly and high-valuation consumers arrive late. Under these premises, it emerges that firms find it optimal to engage in IPD even if there is no uncertainty concerning the arrivals and there is no product differentiation. We model a situation where two firms face a multi-stage Bertrand-Edgeworth competition game. At the first stage, firms simultaneously produce some units of a homogeneous good and in the next stages they sell their production to the different cohorts of consumers. Due to the difficulties in solving the model under this competition scheme, we have found a solution by assuming a simpler price formation mechanism, bases on a fictional auctioneer. Afterwards, we have showed that these results also hold for the

[1] When firms supply non-storable goods, production decision precedes the consumers' purchase and demand is uncertain and fluctuating, firms usually appeal to revenue management techniques (Talluri and van Ryzin, 2005). These methodologies are now diffused in some industries such as airlines, hotels, rental car companies, cruise lines and theatres. The core of revenue management consists of five main elements: forecasting; market segmentation; capacity management; product differentiation; and price discrimination (Botimer, 1996). The use of computer systems allows firms to tackle the complexity of the problem and to implement optimizing and quasi-optimizing methodologies to enhance their revenues. When capacity is costly and marginal costs of production are negligeble, profit maximization coincides to revenue maximization. This explains the use of the term 'revenue management' in place of 'profit management'. Revenue management methodologies are usually implemented by management science and operational research scholars, but are also studied by industrial organization researchers interested in the causes of price dispersion.

Bertrand-Edgeworth game (in some specific cases).

The key element of the model is costly capacity. The intuition behind the result is as follows. As output is not storable and costly, at the first stage of the game, firms limit their quantities. In the next stages, constrained capacity makes interesting for firms to sell a share of their production to low-valuation consumers, in order to have a reduced capacity in the final stages, and to be able to charge high prices to high-valuation consumers. In equilibrium, it emerges that firms equalize the shadow marginal cost of capital with the marginal revenue of each stage.

The model suitably describes the airline industry both in terms of hypotheses and results. In fact, air passengers are usually classified in low-valuation and high-valuation consumers, i.e. leisure and business travellers. Moreover, it is known that business travellers prefer to buy the tickets very close to the departure date, while leisure travellers usually purchase many days before. In terms of results, we find that: (a) firms intertemporal price discriminate and price levels increase in approaching the departure date, (b) the pricing structure in oligopoly mimics that of monopoly case, although business and leisure price levels are lower, (c) in low-demand periods, if firms are not allowed to re-size their planes, they offer discounted fares to very-low-valuation consumers (that usually do not purchase).

The economic literature has devoted much attention to the study of oligopolistic environments when firms can set prices with costly capacity. Bertrand's (1883) model predicted that prices converge to marginal costs when firms compete in prices with unconstrained capacity and product homogeneity. Edgeworth's (1925) model (further analyzed by Dagupta and Maskin, 1986) extended the Bertrand contribution showing that, when firms have capacity constraints, prices diverge from marginal costs, although in general there are no equilibria in pure strategies. Later, Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) demonstrated that capacity precommitment and price competition lead to Cournot outcomes. Davidson and Deneckere (1986) showed that this result depends on the rationing rule chosen by the authors, but they confirmed that capacity constraints, in general, produce prices above the marginal costs. In a more complicated environment, Yanelle (1989) showed that non-competitive outcomes may emerge. Some contributions analyzed the Bertrand-Edgeworth model in a dynamic framework. Dudey (1992) studied when consumers have unit demands and common reservation value. He showed that, contrary to the static case, the model has an equilibrium in pure strategies and firms earn positive profits if their capacity is not too large. Brock and Scheinkman (1985), Benoit and Krishna (1987) and Davidson and Deneckere (1990) analyzed the sustainability of collusive agreements in infinitely repeated game, when capacity is chosen once and for all. Brock and Scheinkman (1985) demonstrated that changes in the number of (capacity constrained) firms have a non-monotone effect on the cartel price. Benoit and Krishna (1987) showed that excess capacity is necessary for collusion and that firms are not able to sustains monopoly prices even if the discounted rate is close to one. Davidson and Deneckere (1990) moving from collusive to semi-collusive equilibria showed that capacity levels and collusion both increase if either interest rates or the cost of capacity fall.

The remainder of the paper is organized as follows. In Section 2, we present the model. Sections 3 and 4 analyze the market-clearing and the Bertrand-Edgeworth competition cases, respectively. Section 5 presents some extensions of the model and some comments on the airline pricing behaviour. Section 6 concludes the paper. To enhance the readability of this paper, all proofs are presented in the Appendix.

2 The model

Assume that Z units of a non-storable good are produced at time 0 and that they will be available at time T. For $t \in \mathcal{T} = \{1, .., T\}$, different cohorts of consumers become potentially interested in buying the good offered on the market. Each cohort can be described by a linear (inverse) demand function:

$$P_t (D_t) = P (r_t, s_t; D_t) = r_t (1 - D_t/s_t), \tag{1}$$

where D_t is the quantity demanded by cohort $t \in \mathcal{T}$, r_t is the the maximal willingness-to-pay of consumers of cohort t, and s_t is the market size of cohort t. There are two firms, named A and B. Firms sustain a cost c for each unit produced at time 0, but zero cost in selling the product at time $t \in \mathcal{T}$. Let be X and Y the production of firms A and B, respectively; and $X + Y = Z$. Firms are free to choose the quantity offered and the price charged at any time t. Let x_t and p_t be, respectively, the quantity and the price offered by firm A at time t. Similarly definitions apply to y_t and q_t for firm B. We make the following assumptions on consumers' behaviour:

1. (Increasing consumer valuation) $r_s < r_t < \infty$, for $s < t$ and $s, t \in \mathcal{T}$.
2. (Efficient rationing rule) Consumers with the highest willingness-to-pay are first to be served.
3. (One visit, at the most) If a consumer belonging to the cohort t is not served, s/he exits the market (i.e. change the date, the destination, the way of transport, stay at home, etc..).

4. (Certainty) The demand is certain.
5. (Viable and unlimited demand for ε prices). $P_T(0) = r_T > c$, $P_1 = \varepsilon$ with $\varepsilon \in (0, c)$, i.e. $r_1 = \varepsilon$ and $s_1 = \infty$.

Assumption 1 implies that business travellers rarely buy tickets many day in advance, while leisure travellers are usually able to purchase many days before the departure date. Assumption 2 is the efficient rationing rule. Usually two explanations are proposed in order to explain this rationing scheme. One is that the most interested consumers are the first who get the ticket (Kreps and Scheinkman, 1983). Alternatively, it is assumed that firms, which charge lower prices, are interested to sell tickets to the high willingness-to-pay consumers so that the opponent remains with the less interesting ones (Osborne and Pitchik, 1986). For this set-up, the first interpretation is preferred, since it does not require firms' strategic behaviour. A discussion on different rationing rules is provided by Davidson and Deneckere (1986). Other rationing schemes (f.e. proportional rationing rule) are at the origin of the price dispersion, e.g. in the Dana (1999)'s model. Assumption 3 is introduced for technical reasons and it will be further discussed in the following. To justify this hypothesis, note that if there is a cost for consumers to visit the store and if they expect increasing prices in time, they will not visit the market a second time. Assumption 4 is clearly unrealistic even if forecasting tools can provide good prediction in many industries. Assumption 5 is for technical reasons. We require that at least one cohort of consumers demand at a price larger than c, and that the first cohort demands infinite quantity for a very small price. In this way firms are sure to sell the all quantity, they produce. To avoid, trivial and unreasonable results $\varepsilon < c$, i.e. it is not profitable to produce just for cohort 1.

We model a game of perfect information considering two different cases.

In case I, the timing of the game is as follows:

- (Consumer demand) Nature determines the demand for each cohort.
- (Capacity choice) At time 0, firms choose X and Y, simultaneously.
- (Allocation choice) At time 0.5, firms choose $(x_t)_{t\in\mathcal{T}}$ and $(y_t)_{t\in\mathcal{T}}$, simultaneously.
- (Pricing game) At time $1, 2, ..T$, firms enter the pricing game, where p_t and q_t are simultaneously determined.

Case I usually concerns multi-market industries, where production often occurs before the consumers' arrivals. Here index t does not refer to time but represents different locations. However, it is also suitable to describe the airline industry, when capacity allocation is not dynamically updated.

Alternatively, in case II, the timing of the game is:

- (Consumer demand) Nature determines the demand for each cohort.
- (Capacity choice) At time 0, firms choose X and Y, simultaneously.
- (Allocation choice for cohort t) At time $t-0.5$, firms choose x_t and y_t, simultaneously.
- (Pricing game for cohort t) At time t, firms enter the pricing game, where p_t and q_t are simultaneously determined.

This setup is more closely related to the current airline industry's behaviour, where capacity allocation is dynamically updated.

We also consider two different versions of the 'pricing game'. The first one is called the market-clearing competition. Similarly to the Cournot case, at time t, firms supply their quantities to consumers and then a fictional auctioneer computes the market-clearing price. In this case: $p_t = q_t = P_t(z_t)$, where $z_t = x_t + y_t$. The second one is the Bertrand-Edgeworth competition. This set-up was originally investigated by Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) for the one period game.

It is worth noting that for asymmetric allocations there is no equilibrium in pure strategies and an equilibrium in mixed strategies occurs, so that a direct computation of the equilibrium is very complex and out of the goals of this paper. Therefore, in the following of the paper, our strategy is to find the equilibrium outcome under market-clearing competition, and then to show that it is also an equilibrium outcome under Bertrand-Edgeworth competition (at least for some special cases).

3 Market-clearing competition

3.1 Monopoly

We start briefly reviewing the monopoly setup, which present many analogies with the duopoly case. Results hold for cases I and II.

Proposition 1 *Under Assumptions 1-5, if capacity Z is given, then, the monopolist charges price $p_t^m = \frac{1}{2}(r_t + \lambda)$, and sells the quantity $z_t^m = \frac{1}{2}\frac{s_t}{r_t}(r_t - \lambda)$ to those cohorts for which $r_t > \lambda$, and supplies nothing to those cohorts for which $r_t \leq \lambda$, where the shadow price of expanding capacity is $\lambda = \left(\sum_{t\in\hat{T}} s_t - 2Z\right)\left(\sum_{t\in\hat{T}} s_t/r_t\right)^{-1}$.*

A formal proof is provided in appendix. In order to clarify the result, notice that the monopolist is free to allocate its capacity among different cohorts. Therefore, s/he will supply those cohorts, which provide larger marginal revenue (MR). Since $MR_1 = P_1 = \varepsilon > 0$, the firm can gain, at least, ε from each unit of capacity.

There are two notable cases depending on the capacity size. When capacity is small, since $MR_t(0) = P_t(0) = r_t$, the monopolist will supply only those segments with the highest willingness-to-pay. Being the monopolist free to move capacity from one cohort to the other, marginal revenue will be equalized among the active cohorts. This yields:

$$MR_t(z_t) = \lambda \text{ for those } t \text{ such that } r_t > \lambda,$$

where λ is the shadow price of expanding capacity, i.e. the price that the monopolist would pay for having an additional unit of capacity. Since marginal revenue is decreasing in z_t, then the larger is Z and the lower will be λ.

If the monopolist can choose the size of the plane, it will choose the capacity level, for which $c = \lambda$. Therefore:

Corollary 1 *Under Assumptions 1-5, a monopolist sets capacity equals to* $Z^M = \sum z_t^M$. *It charges price* $p_t^M = \frac{1}{2}(r_t + c)$, *and sells quantity* $z_t^M = \frac{1}{2}\frac{s_t}{r_t}(r_t - c)$, *to those cohorts for which* $r_t > c$, *and supplies nothing to those cohorts for which* $r_t \leq c$.

3.2 Duopoly

When we move to a strategic environment, it is important to specify which information is available to a firm, when it decides on capacity allocation. In case I, a firm, when chooses how to allocate capacity, has information only on the overall capacity supplied by the opponent; therefore it conditions its choice only on time. In difference games' vocabulary, we say that the firm is playing an open loop strategy. In case II, a firm has information not only on the overall capacity, but also on the capacity allocation of the rival (in previous periods). In this case, it seems natural to assume that the firm can use this information to condition their strategy. In particular, we assume that in any period, the firm bases its strategy on its and opponent's residual capacities and not on the overall history.[2] Therefore, in this case, we say that the firm plays a closed-loop strategy (or feedback strategy).

We start to solve the model by assuming market-clearing competition. As in the Cournot model, we have no realistic price-setting mechanism, so that we imagine a fictional auctioneer

[2] In this paper, we focus on the Markovian strategies, i.e. those strategies based only on the current situation (i.e. the residual capacity), but not on the full history (i.e. how the capacity has been allocated in previous periods). The use of Markovian strategies is a natural choice in this setup as in case II, the history of the game till a particular time can be summarized by the value of the state variable.

who sells the total supply by searching for a price that exactly clears the market.

Let $(\hat{x}, \hat{y})$ be an equilibrium, if firms A and B have given capacities X and Y, where $\hat{x} = (\hat{x}_1, .., \hat{x}_T)$ and $\hat{y} = (\hat{y}_1, .., \hat{y}_T)$. The following lemmas help us to characterize the firms' equilibrium outcome. Lemma 1 shows that firms will want to serve consumers of the richest cohorts, while Lemma 2 implies that firms with larger capacity will supply a larger quantity to each cohort, and to a larger number of cohorts.

Lemma 1 *In case I, for $\hat{t} \neq T$, if $\hat{x}_{\hat{t}} > 0$ then $\hat{x}_{\hat{t}+1} > 0$; (similarly, if $\hat{y}_{\hat{t}} > 0$ then $\hat{y}_{\hat{t}+1} > 0$).*

Lemma 2 *In case I, if $X \geq Y$ then $\lambda_x \leq \lambda_y$ and $x_t \geq y_t$ for every t; and $t_A \leq t_B$, where $t_A = \min_t \{t : x_t > 0\}$, $t_B = \min_t \{t : y_t > 0\}$.*

Previous results rest on the fact that to be an equilibrium it is necessary that firms equalize their marginal revenue in each market they participate, i.e.:

$$\begin{aligned} MR_t^A (x_t + y_t) &= \lambda_x \text{ for those } t \text{ such that } MR_t^A (y_t) > \lambda_x, \\ MR_t^B (x_t + y_t) &= \lambda_y \text{ for those } t \text{ such that } MR_t^B (x_t) > \lambda_y. \end{aligned}$$

Since in the first market, a firm can gain ε, for every X and Y, it follows that: $\lambda_x, \lambda_y \geq \varepsilon$. Comparing Proposition 1 and Lemmas 1 and 2, we note that the optimal strategies both in monopoly and in duopoly are similar. When capacity is large with respect to demand, firms will offer to some customers discounted tariffs, while when capacity is small, firms want to offer their products only to the most interested consumers.

Corollary 2 *For any X and Y, equilibrium prices are increasing in t.*

Note that Corollary 2 ensures that if a consumer belonging to the cohort t is not served, s/he will not buy in the next periods. This means that the restriction of 'only one visit' is a result of the model and thus, Assumption 3 is not strictly necessary for the analysis.

Proposition 2 describes the equilibrium when capacities are given.

Proposition 2 *Under Assumptions 1-5, if firms A and B have given capacities $X, Y > 0$ with $X \geq Y$, and they play open-loop (case I) or closed-loop strategies (case II), then the* *capacity allocation and prices are given by:*

$$x_t = \begin{cases} \frac{1}{3}\frac{s_t}{r_t}\left(r_t - 2\lambda_x + \lambda_y\right) & t \geq t_B \\ \frac{1}{2}\frac{s_t}{r_t}\left(r_t - \lambda_x\right) & t_A \leq t < t_B \\ 0 & t < t_A \end{cases}, \tag{2}$$

$$y_t = \begin{cases} \frac{1}{3}\frac{s_t}{r_t}\left(r_t - 2\lambda_y + \lambda_x\right) & t \geq t_B \\ 0 & t < t_B \end{cases}, \tag{3}$$

$$p_t^c = q_t^c = \begin{cases} \frac{1}{3}\left(\lambda_x + \lambda_y + r_t\right) & t \geq t_B \\ \frac{1}{2}\left(r + \lambda_x\right) & t_A \leq t < t_B \\ p & t < t_A \end{cases}, \tag{4}$$

$$\lambda_x = \left(S_1 + S_2 - 2X - Y\right) / \left(R_1 + R_2\right) \text{ and} \tag{5}$$

$$\lambda_y = \left(2\left(S_2 - X\right) R_2 + \left(S_2 - 3Y\right) R_1 + \left(S_1 - 4Y\right) R_2\right) / \left(2R_2^2 + 2R_1 R_2\right), \tag{6}$$

where $\mathcal{T}_A = \{t_A, .., T\}$, $\mathcal{T}_B = \{t_B, .., T\}$, $S_1 = \sum_{t \in \mathcal{T}_A \setminus \mathcal{T}_B} s_t$, $S_2 = \sum_{t \in \mathcal{T}_B} s_t$, $R_1 = \sum_{t \in \mathcal{T}_A \setminus \mathcal{T}_B} \frac{s_t}{r_t}$ *and* $R_2 = \sum_{t \in \mathcal{T}_B} \frac{s_t}{r_t}$.

When firms can choose their capacities, the following result emerges.

Corollary 3 *Under Assumptions 1-5, when firms can freely choose X and Y, then:*

$$x_t^C = y_t^C = \begin{cases} \frac{1}{3}s_t\left(1 - c/r_t\right) & \text{if } r_t \geq c \\ 0 & \text{if } r_t < c \end{cases}, \tag{7}$$

$$p_t^C = q_t^C = \begin{cases} \frac{1}{3}\left(2c + r_t\right) & \text{if } r_t \geq c \\ 0 & \text{if } r_t < c \end{cases}, \tag{8}$$

and $X^C = Y^C = \sum_{t=1}^{T} x_t^C$.

Assume that capacities are already given. To be an equilibrium, firms want to allocate the capacity in such a way to equate their marginal revenue in each market. Hence, in the investment decision, the capacity choice is obtained by equating the marginal revenue to the marginal cost. Market-clearing competition produces a situation in which marginal revenue is c (the shadow cost of capital) for every cohort of consumers, and coincides with marginal costs. In this case, firms have no incentives to increase their offer to a cohort and to reduce their offer to another. Hence, firms choose the Cournot quantities for each cohort of consumers. This setup is similar to the multi-market game described by Anderson and Fischer (1989). As they noted, the emergence of Cournot outcomes relies on the hypothesis of linear demand. Assuming different functional forms usually yield to different results. Anderson and Fischer (1989) have showed that a deviation from the Cournot capacity occurs when two simultaneous conditions realize: first, firms 'wish' to modify capacities, and, second, firms have the 'ability' to induce a change in the quantity supplied by the opponent. In our setup, the first condition is satisfied since some markets are more profitable than others ($r_t \neq r_s$ with $t \neq s$), but the second condition does not hold due to the linearity of demand.

4 Bertrand-Edgeworth competition

In this section we provide an explicit model of price formation without recurring to the auctioneer. However, the use of Bertrand-Edgeworth competition reduces the tractability of the model. Even in the static case, when capacities are not symmetric, there is no equilibrium in pure strategies but only in mixed strategies. Additional difficulties come from the fact that even with linear demand, the marginal revenue is not only discontinuous but also not monotonic. Dasgupta and Maskin (1982) establish the existence for the case of linear demand and constant marginal costs. Davinson and Deneckere (1990) computed the expected revenue for the linear demand with $r = s = 1$. In Appendix, we present the same results for generic values of r and s. In a single period game ($T = 1$), the market clearing competition as well as the Bertrand-Edgeworth competition yields to the same results, i.e. the Cournot outcome (Kreps and Scheinkman, 1983): $x = y = s\,(1 - c/r)\,/3$. When $T = 2$, thanks to Assumption 5, the equilibrium outcome is the same of Cournot.

Proposition 3 *For cases I and II, under assumptions* $1 - 5$*, when* $T = 2$*, the equilibrium outcomes of the Bertrand-Edgeworth competition and of the market-clearing competition coincide.*

To solve the problem in a more general situation, we start from the solution of the market-clearing competition and we show that it is a solution also under Bertrand-Edgeworth competition. We provide a proof of the equivalence of the pricing strategies in two, even not perfectly satisfactory, ways. The first result is obtained by assuming an equilibrium concept weaker than the Nash, i.e. the local Nash equilibrium. Second, the equivalence is tested by simulation for a wide range of parameters. In both cases, we verify that when firm A has chosen X^C, firm B has no incentive to change its capacity from Y^C and that outcomes of the game coincide with those obtained under the market-clearing competition. With this procedure is not possible to guarantee that the equilibrium is unique.

4.1 Local Nash equilibrium

Rothschild and Stiglitz (1976) introduced the concept of local Nash equilibrium. A solution profile is a local Nash equilibrium if players have no incentive to unilaterally deviate in the nearness of the solution profile. Thus, the equilibrium is resistant to small deviations, but not necessarily to large ones. In words, a local Nash equilibrium is a strategy profile such that no player has an incentive to deviate to a similar close strategy. This implies that each player is using a local maximum of his payoff function given the strategies of other player.[3]

Proposition 4 *For cases I and II, under assumptions* $1-5$*, the equilibrium outcome of the market-clearing competition is a local equilibrium outcome of the game under Bertrand-Edgeworth competition.*

4.2 Simulation

We have also investigated the existence of global Nash equilibria. By simulation, we find that for the parameters' range investigated local equilibria are also global. Simulated results are confined to the case $T = 2$ a and we have removed Assumption 5 in order to have two standard linear demand functions. We investigate the equilibrium only in case I.[4] In particular, we have computed the optimal values of x_t, y_t, X and Y by Corollary 3, for some values of c, s_t and r_t, and then we have tested whether firm B has an incentive to deviate from the equilibrium providing a larger or a smaller capacity. For each capacity choice of B, we have find the optimal allocation for A and B. The result is obtained by modifying the capacity Y from 0 to the double of the Cournot solution, and then computing the equilibrium using (9).

We have chosen the following parameters: $c = 0.1$, $r_1, r_2 \in [1, 3]$ and $s_1, s_2 \in [1, 3]$. We have also done random proofs for different parameters values all bringing to additional confirmation of the equivalence result. By simulation, we have positive conclusions that the outcome under the market clearing competition produce a similar outcome in the Bertrand-Edgeworth competition, even if we are not able to infer that it is the unique equilibrium.

[3] For a discussion on the existence of a global Nash equilibrium in pure strategies, see: Zied (2003). Bonanno (1996), Sened (1996). Schofield and Sened (2002) provide a brief justification on the use of a local Nash equibrium in games. Alós-Ferras and Ania (2001) provide a formal definition of local Nash equilibrium.

[4] Although the two situations seem similar since by choosing X and x_1 the capacity allocated is automatically determined: $x_2 = X - x_1$; however, since in the Bertrand-Edgeworth competition the quantity sold does not necessarily coincides with the quantity charged, the simulation results only refer to the case I, where the unsold capacity of the first period can not be offered in the second one.

By simulation, we have also computed the capacity allocations using capacity choices, which differ from Cournot outcome. As expected, when the capacity is smaller than Cournot outcome, the equilibrium is that proposed in Proposition 2, while when there are large capacities for one or both carriers, the Edgeworth-Bertrand outcome substantially differs from the market-clearing equilibrium and asymmetric and multiple allocations emerge. The explanation is quite simple. See, Equation (9). When capacities of both firms in the same market are large (Region D), then firms behave as in a Bertrand game, so that they realize zero profits. To do better, firms have to choose asymmetric allocations, which produce positive returns, even if smaller than the Cournot ones.

The result changes when re-introducing Assumption 5. In fact, when capacities are large, carriers sell a share of their production to low-valuation consumers at low prices in order to be able to supply Cournot quantities to high-valuation consumers.

5 Pricing behaviour of airline carriers

We now use the results presented in Sections 3 and 4 to interpret the behaviour observed in the airline industry.

5.1 Pricing behaviour in practice

As widely investigated, pricing behaviour in legacy and low cost carriers present many differences but also some analogies. Legacy carriers organize seats into reservation classes. Each class refers some product characteristics, called ticketing restrictions (such as cabin, priority check-in, ticket refundability, advance purchase restrictions, valid travel days, or stay restrictions). Usually, classes are hierarchically organized, meaning that they are ordered from the highest fare to the lowest one. When carriers decide capacity allocation they usually use a nested system. They reserve x_1 seats for class 1 (i.e. the lowest class), $x_2 > x_1$ seats for class 1 and 2 jointly, and so on and so forth. When the tickets sold to class 2 exceed $x_2 - x_1$, then seats available for class 1 decrease accordingly. When the available seats for a class end, the class is closed. Alternatively, less sophisticated management strategies allocate capacity to separate classes. Consumers, when buying a ticket, faces different fares which correspond to different classes. The use of classes allows carriers to implement both intertemporal and product differentiation simultaneously. Selling tickets with identical characteristics implies that only lowest available class is bought, but due to product differentiation travellers can also be interested in paying higher fares. Our model assumes that product is homogeneous so that consumers will always pay the cheapest available fare. When carriers set the capacity once and for all, we are in Case I, while when capacity can be adjusted we are in case II of our model. Clearly, dynamic capacity allocation can be a response to the competitor behaviour or to cope with unforecast demand variations. This last aspect is not captured by the model, which assume certain demand.

The low-cost carriers' pricing behaviour is quite different from that of legacy carriers as they employ a price setting procedure based on the departure date and on the number of occupied seats. Usually, they increase the price in the period approaching the departure date and depending on the observed load factor. Product, in this case homogeneous, is so that low-cost carriers' are only involved in temporal product discrimination.

The main analogy between the two pricing behaviours is on the fact that in both cases carriers' decisions is a price-quantity choice (as in the Edgeworth-Bertrand model).

5.2 Price setting: monopoly vs oligopoly

The main effect of (simultaneous) Bertrand-Edgeworth competition is that there is no marginal cost pricing, even if firms compete in price, since capacity is limited.

Without capacity constraints, under Assumptions 1-5, the monopoly pricing behaviour remains that predicted in Corollary 1, but duopoly pricing would be $p_t = q_t = c$ for every cohort of consumers, a result quite different from that observed in real markets. Limited capacity, indeed, allows for explaining similar pricing pattern emerging both in monopoly and in duopoly environments. In table 1, we show the average fares per class of service in monopoly and duopoly, on two-way international flights in Europe in 2004 (based on a sample of 15 O-D destinations and the 4 main European legacy carriers) computed in Alderighi et al. (2004).

Table 1: Average fares per class of service (euros).

Class of service	Monopoly	Duopoly
Promotional	183	169
Discounted1	305	266
Discounted2	395	350
Economy1	490	441
Economy2	607	557
Unrestricted1	893	780
Unrestricted2	977	883

(Alderighi et al., 2004)

The pricing patterns for monopoly and duopoly are similar, although monopoly fares are higher the duopoly ones.[5] The result is qualitative similar to those predicted by Corollary 1 and Corollary 3. In particular, since lower classes (Promotional, Discounted) are closed before the others (Economy, Unrestricted), then prices are increasing in approaching the departure date (Lemma 1).

But, why carriers will sell tickets to cohorts of consumers with lower willingness-to-pay without waiting for higher-valuation consumers? The result rests on the type of competition. If carriers remain with large capacity for higher classes, they will not able to charge high fares to these cohorts of consumers, since pricing competition produces Bertrand-like results. On the contrary, by allocating part of their capacity to the lower classes, they are able to remain with the right capacity to obtain a Cournot-like result.

In Borenstein's (1985) and Holmes' (1989) papers, it is assumed that there are different consumers' groups and products are differentiated (different departure date, service, etc..), so that carriers can segment consumers on the basis of the elasticity market demand (monopoly-type discrimination) and on cross-elasticity of demand (competitive-type discrimination). In this setup, we have assumed that products are homogeneous so that competitive-type discrimination is not possible. This fact explains the reason why, in this setup, price dispersion is larger in monopoly than in duopoly.

5.3 Systematic peak-load pricing

Until now, we have assumed that firms are able to choose the 'Cournot' capacity for a given demand level and theoretical pricing patterns match the observed ones. Quite interestingly, Proposition 2 is useful to show that firms, even if capacities are set once and for all, can sustain a pricing pattern that is similar to empirical observations.

The reasoning is analogous to the argument presented in previous paragraph. When demand is low with respect to capacity, since the shadow cost of a seat is small, carriers will allocate capacity to discount classes, to be able to set high fares to high-evaluation consumers and to not incur in the Bertrand trap. When demand is high, carriers will not allocate capacity to discount classes but only to high classes.

This effect, previously identified by Borenstein and Rose (1994), was called 'systematic peak-load pricing'.

[5] Similar results concerning the low cost carriers are presented in Bachis and Piga (2006).

6 Conclusions

This paper has presented a simple model of pricing that allows to explain some regularities observed in the airlines behaviour: (a) firms intertemporal price discriminate and price levels increase in approaching the departure date, (b) the pricing structure in oligopoly mimics that of monopoly case, although business and leisure price levels are lower, (c) in low-demand periods, if firms are not allowed to re-size their planes, they offer discounted fares to very-low-valuation consumers (that usually do not purchase).

It show that these results are due to the nature of competition. Even if firms compete in prices, capacity constrains limit the Bertrand-like outcomes, making the model predictions in line with Cournot setup.

This paper also provides a justification of the use of revenue management techniques (usually developed in monopoly) in oligopolistic markets, by making the role of capacity explicit.

The model is based on many ad hoc assumptions, and some of these have important effects on the results.[6] However, although conscious that the dynamic competition probably produces different results from those of Cournot, we think that this paper provides some insights on the eventuality that Cournot-like results may occur.

7 Appendix

7.1 Bertrand-Edgeworth pricing game

This result is directly derived from Davidson and Deneckere (1990). Demand is given by $P(r,s;q) = r(1-D/s)$, and capacity supplied by firms are x and y. For each pair (x,y) the one-shot price-setting game with capacity constraints has a unique static Nash equilibrium given by (9):

$$R_X(r,s;x,y) = \begin{cases} rx(s-(x+y))/s & (x,y)\in A \\ \frac{1}{4}r(s-y)^2/s & (x,y)\in B_X \cup C_X \\ rx(s-x)^2/(4sy) & (x,y)\in B_Y \\ \frac{1}{2}rx\left(s-\sqrt{x(2s-x)}\right)/s & (x,y)\in C_Y \\ 0 & (x,y)\in D \end{cases}, \tag{9}$$

[6] For example, the linearity of demand function makes the static and the dynamic Cournot game outcomes identical limiting the strategic effect of increasing capacity (Anderson and Fischer, 1989); or the efficient rationing rule assumption permits the Cournot outcome of the Bertrand-Edgeworth game, while under other rationing rules it may differ (Davidson and Deneckere, 1986).

where $A = \left\{(x,y) : y \leq \frac{1}{2}(s-x) \ \wedge \ x \leq \frac{1}{2}(s-y)\right\}$, $B_X = \left\{(x,y) \notin A : \ x \leq \frac{1}{2}\left(s + \sqrt{y(2s-y)}\right) \wedge \ x \geq y\right\}$, $B_Y = \left\{(x,y) \notin A : \ y \leq \frac{1}{2}\left(s + \sqrt{x(2s-x)}\right) \wedge x < y\right\}$, $D = \{(x,y) : x, y \geq s\}$, $C_X = \{(x,y) \notin A \cup B_X \cup D, \, x > y\}$, and $C_Y = \left\{(x,y) \in \Re_+^2 : (x,y) \notin A \cup B_Y \cup D, \, y > x\right\}$.

And similarly for firm B.

7.2 Proofs

Proof. of Proposition 1. (Case I) The monopolist faces the following problem:

$$\max_{\{z_1,..,z_T\}} \sum_{t \in \mathcal{T}} P_t(z_t) z_t \text{ s.t. } Z \geq \sum_{t \in \mathcal{T}} z_t \text{ and } z_t \geq 0. \tag{10}$$

We form the Lagrangian: $\mathcal{L} = \sum_{t \in \mathcal{T}} P_t(z_t) z_t - \lambda \left(\sum_{t \in \mathcal{T}} z_t - Z\right) - \mu_t z_t$

First order conditions imply that: $r_t(1 - 2z_t/s_t) = \lambda + \mu_t$. Hence: $z_t = \frac{1}{2}\frac{s_t}{r_t}(r_t - \lambda - \mu_t)$. Note that $\mu_t = 0$, when $z_t > 0$; and $\mu_t > 0$ when $\lambda < r_t$. Define $\hat{t} = \min_t \{t : z_t > 0\}$. From Assumption 1, for $\hat{t} \neq T$, $z_{\hat{t}} > 0$ implies $z_{\hat{t}+1} > 0$. Define $\hat{\mathcal{T}} = \left\{\hat{t}, .., T\right\}$. Since, $Z = \sum_{t \in \hat{\mathcal{T}}} z_t$, then: $\lambda = \left(\sum_{t \in \hat{\mathcal{T}}} s_t - 2Z\right)\left(\sum_{t \in \hat{\mathcal{T}}} s_t/r_t\right)^{-1}$. Assumption 5 ensures that $\lambda > 0$, and therefore all the capacity is allocated. Finally, $p_t = P_t(z_t) = \frac{1}{2}(r_t + \lambda)$ if $t \in \hat{\mathcal{T}}$. By the contrary, the monopolist is free to charge whatever price to cohorts, which are not served.

(Case II) The solution is the same of Case I. To show this note that, the maximization problem in (10) is equivalent to:

$$\max P(Z_t - Z_{t-1})(Z_t - Z_{t-1}) - \lambda (Z_t - Z_{t-1}),$$

$$\text{s.t. } Z_1 = 0, \, Z_T \geq 0, \, Z_{t+1} = Z_t - z_t \text{ and } (Z_t - Z_{t-1}) \geq 0,$$

which has the same solution of the previous problem. ■

Proof. of Lemma 1. (Case I) First note that, in general, $MR_t^A(x_t + y_t) \gtreqless MR_t^B(x_t + y_t) \Leftrightarrow x_t \lesseqgtr y_t$. We prove it by contradiction. Assume that exists another equilibrium, namely $(\tilde{x}, \tilde{y})$, such that $\tilde{x}_t > 0$ and $\tilde{x}_{\hat{t}} = 0$. It means that:

$$MR_t^A(\tilde{x}_t + \tilde{y}_t) > MR_{\hat{t}}^A(\tilde{y}_{\hat{t}}), \tag{11}$$

and hence $\tilde{y}_{\hat{t}} > 0$. Now, if $\tilde{y}_t = 0$ then $MR_t^B > MR_{\hat{t}}^B$ and thus $\lambda_y > \lambda_x$. But it is a contradiction as from the assumption $MR_t^A > MR_{\hat{t}}^A(\tilde{y}_{\hat{t}})$ and thus $\lambda_x > \lambda_y$. Now consider the case where $\tilde{y}_t > 0$, and hence:

$$MR_t^B = MR_{\hat{t}}^B. \tag{12}$$

Subtracting the double of (12) from (11) and using (1), we obtain: $r_t(1 - 3\tilde{x}_t/s_t) \geq r_{\hat{t}}$, which is a contradiction.

■

Proof. of Lemma 2. Straightforward. ■

Proof. of Proposition 2. (Case I) Firm A faces the following problem (and similarly for B.):

$$\max_{\{x_1,..,x_T\}} \sum_{t\in\mathcal{T}} P_t\left(x_t+y_t\right)x_t \text{ s.t. } X \geq \sum_{t\in\mathcal{T}} x_t \text{ and } x_t \geq 0. \tag{13}$$

We form the Lagrangian:

$$\mathcal{L} = \sum_{t\in\mathcal{T}} P_t\left(x_t+y_t\right)x_t - \lambda_x\left(\sum_{t\in\mathcal{T}} x_t - X\right) - \mu_t x_t$$

First order conditions imply that: $r_t\left(1-\left(y_t+2x_t\right)/s_t\right) = \lambda + \mu_t$. Hence: $x_t = \frac{1}{2}\frac{s_t}{r_t}\left(r_t - \lambda - \mu_t\right) - \frac{1}{2}y_t$. Note that $\mu_t = 0$, when $x_t > 0$; and $\mu_t > 0$ when $\lambda_x < P\left(y_t\right)$. From Lemma 1, it is easily to show that $t_A \leq t_B$. Since, $X = \sum_{t\in\mathcal{T}_A} x_t$, and $Y = \sum_{t\in\mathcal{T}_B} Y_t$ then:

$$\begin{aligned} \frac{1}{3}S_2 - \frac{2}{3}\lambda_y R_2 + \frac{1}{3}\lambda_x R_2 &= Y \\ \frac{1}{2}S_1 - \frac{1}{2}\lambda_x R_1 + \frac{1}{3}S_2 - \frac{2}{3}\lambda_x R_2 + \frac{1}{3}\lambda_y R_2 &= X \end{aligned}$$

Solving the system for λ_x and λ_y yield (5) and (6). Assumption 5 ensures that λ_x and $\lambda_y > 0$, and therefore all the capacity is allocated. Finally, $p_t = P_t\left(x_t+y_t\right)$.

(Case II) The solution is the same of Case I. To show this note that, the maximization problem in (13) is equivalent to:

$$\max P\left(X_t - X_{t-1} + y_t\right)\left(X_t - X_{t-1}\right) - \lambda\left(X_t - X_{t-1}\right),$$

$$\text{s.t. } X_1 = 0,\ X_T \geq 0,\ X_{t+1} = X_t - x_t \text{ and } \left(X_t - X_{t-1}\right) \geq 0.$$

■

Proof. of Proposition 3. Backward induction can be applied to find the outcome of the game, as there is perfect information. The analysis of the sub-game starting after capacity choice is similar to that of Kreps and Scheinkman (1983) by replacing the capacity choice of the static game with the allocation choice for the second cohort. Payoffs of A are $\varepsilon\left(X - x_2\right) + R_X\left(r, s; x_2, y_2\right)$ and analogously for B. Since in the first market, the marginal revenue is lower than the marginal cost, firms can be induced to install a capacity larger than the Cournot quantity only if it has a strategic effect on the behaviour of the opponent. Not this the case, the outcome of the game is the Cournot one. ■

Proof. of Proposition 4. Provided that $c > 0$ then looking at equation (2), it emerges that for small deviations from the equilibrium, the allocations remains in region A. This means that the game is in the same situation of the market-clearing competition, then the problem is the same as that described in Corollary 3. ■

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